٢٠١٧/٣/١ lec 4

* Electric and Magnetic Potential vectors $\bar{V}$ and $\bar{A}$

① Static Case (NoT Time Varying)

اثبات for electric Potential vector $\bar{V}$

* The relation between the electric field $\bar{E}$ and the electric Potential vector $\bar{V}$ is given by

① $\bar{E} = -\nabla \bar{V} \Rightarrow \frac{dV}{dx}\hat{x} + \frac{dV}{dy}\hat{y} + \frac{dV}{dz}\hat{z}$

* $\bar{D} = \epsilon \bar{E}$

* $\nabla \cdot \bar{D} = \rho_v \Rightarrow$ Source

② * $\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon}$

بالتعويض من ① و ②

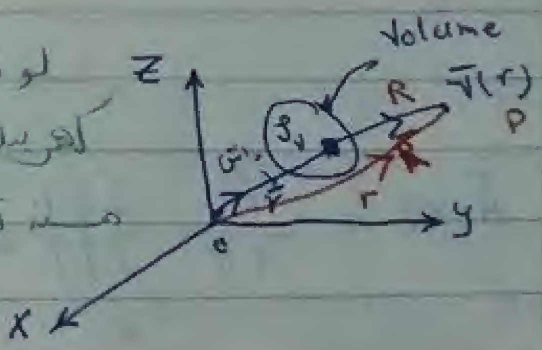
* $\nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$

③ * $\boxed{-\nabla^2 \bar{V} = \frac{\rho_v}{\epsilon}} \Rightarrow$ Poisson's equation *

حلها.

* $\bar{V}(r) = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho_v(r')}{R}\, dv \Rightarrow$ (A) معادلة الجهد الكهربى

داش



لو فرضنا وجود Volume موزع عليه شحنة
كهربية مقدارها ρ_v وموجود على مسافة r'
من نقطة الاصل فانه سوف يولد جهد كهربى
مقداره $\bar{V}(r)$ على مسافة (R) من الجسم
وعلى مسافة (r) من نقطة الاصل

$\Rightarrow \rho_v \rightarrow V(r) \rightarrow \bar{E} = -\nabla \bar{V}$

بالتكامل

$\bar{E} \rightarrow \bar{H} \rightarrow P$

* Magnetic Potential vector $\bar{A}$

* 1 $\nabla \cdot \bar{B} = 0$

* 2 $\nabla \cdot (\nabla \times \bar{A}) = 0$

بمقارنة ① ، ②

$$\bar{B} = \nabla \times \bar{A}$$

$$\bar{B} = \mu \bar{H}$$

* 3 $\bar{H} = \frac{1}{\mu} (\nabla \times \bar{A})$

4 $\Leftarrow \nabla \times \bar{H} = \bar{J} \rightarrow$ source

بأخذ ∇ ③

$$\nabla \times \bar{H} = \bar{J} = \frac{1}{\mu} (\nabla \times \nabla \times \bar{A})$$

$$\frac{1}{\mu} (\nabla \times \nabla \times \bar{A}) = \bar{J}$$

العديد بعض بحطه باسف

$$\cancel{\nabla (\nabla \cdot \bar{A})} - \nabla^2 \bar{A} = \mu \bar{J}$$

كمية قياسية تفاضل

$= 0$

$$\boxed{-\nabla^2 \bar{A} = \mu \bar{J}} \Rightarrow$$ معادلة بواسون

حله.

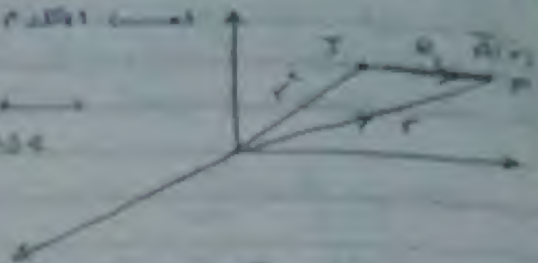
$$\boxed{\bar{A}(r) = \frac{\mu}{4\pi} \iiint_{v} \frac{\bar{J}(\bar{r})}{R} \, dv}$$

Magnetic potential

Solve equation for

A + V static case

* Time varying case



$$\bar{J}(r') \xrightarrow{\int} \bar{A}(r) \xrightarrow{\frac{d}{dr}} \bar{H} = \frac{1}{\mu}(\nabla \times \bar{A})$$

* Time Varying Case *

$$\bar{E}(r,t) = \underset{\text{Mag.}}{\bar{E}(r)}\ \underset{\text{phase}}{e^{j\omega t}}$$

$$\bar{H}(r,t) = \bar{H}(r) \cdot e^{j\omega t}$$

$$\rho_v(r,t) = \rho_v(r)\, e^{j\omega t}$$

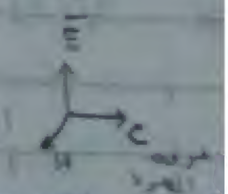
$$\bar{J}(r,t) = \bar{J}(r)\, e^{j\omega t}$$

$$\bar{A}(r,t) = A(r)\, e^{j\omega t}$$

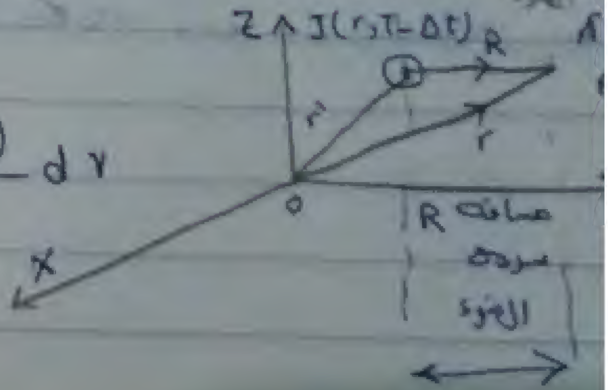
$$\bar{V}(r,t) = V(r)\, e^{j\omega t}$$

Case ① $\bar{A}$

$$\Rightarrow \bar{A}(r,t) = \frac{\mu}{4\pi} \iiint_V \cancel{\frac{\bar{J}(r',t)}{R}}\, dv$$



$$\bar{A}(r,t) = \frac{\mu}{4\pi} \iiint_V \frac{\bar{J}(r', t-\Delta t)}{R}\, dv$$



* $\bar{A}(r, t)$ ، Time varying ، Drive expression

$\bar{A}$ عند اللحظة (t) ناتج عن تيار (J) والموجة الكهرومغناطيسية
في اللحظة $(t-\Delta t)$ ما قطعت مسافة (R) بسرعة الضوء حتى
وصلت عند اللحظة (t)

$$\Delta t = \frac{R}{C} \quad \frac{\text{مسافة}}{\text{سرعة}}$$

$$\bar{A}(r)\, e^{j\omega t} = \frac{\mu}{4\pi} \iiint_V \frac{\bar{J}(r')\, e^{j\omega t}\, e^{-j\omega \Delta t}}{R}\, dv$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V J(r')\, \frac{e^{-j\omega \Delta t}}{R}\, dv$$

$$\omega \Delta t = \omega \frac{R}{c}$$

$$= \frac{2\pi f}{c} R = \overset{B}{\frac{2\pi}{\lambda}} R \qquad B = \frac{2\pi}{\lambda}$$

$$= BR$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V J(r')\, \frac{e^{-jBR}}{R}\, dv$$

نحفظ

$$* \quad \bar{V}(r) = \frac{1}{4\pi \epsilon} \iiint_V \rho_v(r')\, \frac{e^{-jBR}}{R}\, dv$$

لو طلب الاثبات للمعادلات دي #

① نبدأ بالاثبات من اول static case وبعد كده ندخل ال Time varying ونستبدل

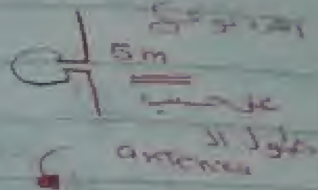
[1] Dipole antennas

* are wire antennas that can be classified according to their lengths, they are classified into 3 categories ⟹

(1) Elementary dipole (infinitisimal) $\Delta L < \frac{\lambda}{100}$

[2] Short dipole $\frac{\lambda}{100} < \Delta L < \frac{\lambda}{10}$

[3] long dipole $L > \frac{\lambda}{10}$



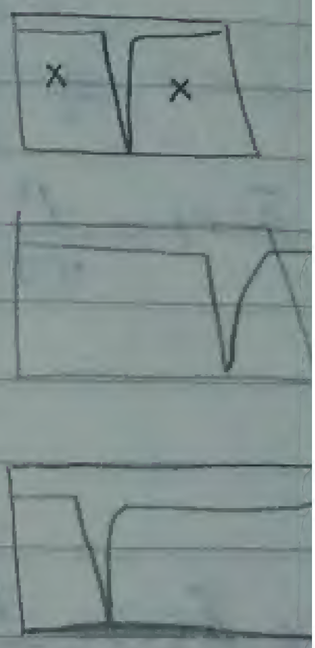
* ملحوظة: في حل اى مسائل لازم التأكد من طول ال antenna لتحديد نوعها

[1] Elementary dipole

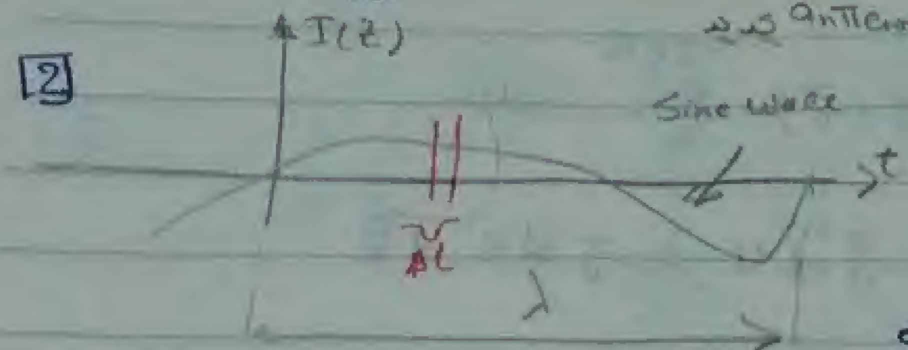
(1) $\Delta L < \frac{\lambda}{100}$

ملحوظ
طول ال antenna يحدد

$S_{11} = \Gamma$



[2]



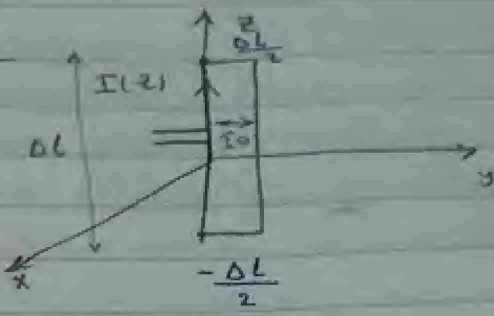
ملحوظة

* طول ال antenna هو الذى يحدد ال Current distribution

*(2) The current distribution accross the antenna is constant.

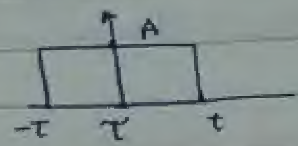
في resonant circuit

$$I(z) = \begin{cases} I_o & -\frac{\Delta l}{2} \le z \le \frac{\Delta l}{2} \\ 0 & o.w \end{cases}$$



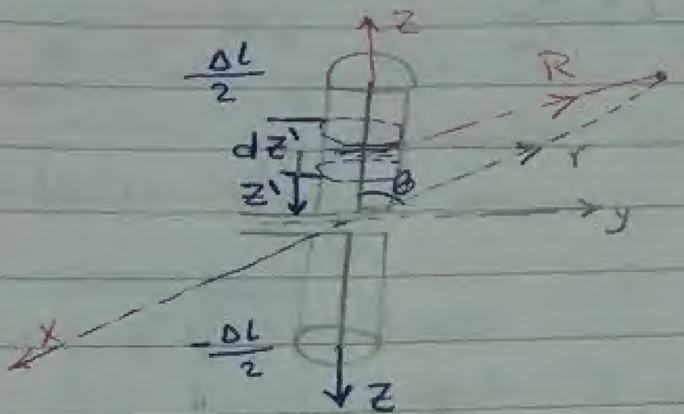
rect w.r.t

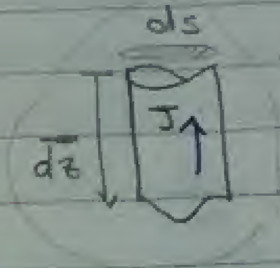
$$\boxed{I(z) = I_o \,\text{rect}\, \frac{z}{\Delta l}} \;\#$$



$A \,\text{rect}\left(\frac{t}{\tau}\right)$

* Derive an expression for $\bar{A}$





$dv = ds\, d\bar{z}$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V J(\bar{z}) \frac{e^{-j\beta R}}{R}\, dv$$

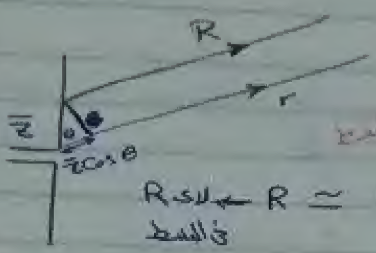
$$\bar{J}\, dv = J\, ds\, d\bar{z}$$

$$= I(z')\, d\bar{z}$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(z') \frac{e^{-j\beta R}}{R}\, dz'$$

* at far field

* approximation



الـ R في الأس البسط

الـ R في البسط ← $R \simeq r - \bar{z}\cos\theta \longrightarrow (1)$

في المقام الـ R والمقام $\dfrac{1}{R} \simeq \dfrac{1}{R} \longrightarrow (2)$

$$\bar{A}(r) = \frac{\mu}{4\pi} \int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(z') \frac{e^{-JB(r - z'\cos\theta)}}{r} \, d\bar{z}$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \frac{e^{-JBr}}{r} \int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(\bar{z}) \, e^{J(B\cos\theta)z'} \, dz'$$

let $\boxed{B\cos\theta = \omega}$ $\qquad f(\omega) = \int_{-\infty}^{\infty} f(t) \, e^{J\omega t} \, dt$

$$\bar{A}(r) = \underbrace{\frac{\mu}{4\pi} \frac{e^{-JBr}}{r}}_{\text{ثابت}} \underbrace{\int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} I(z') \, e^{J\omega\bar{z}} \, dz'}_{FT}$$

FT of $I(z')$ تقريبا يكافئ

$$\bar{A}(r) = \frac{\mu}{4\pi} \frac{e^{-JBr}}{r} FT[I(z')]$$

$$I_o \, rect\left(\frac{z}{\Delta l}\right) \xrightarrow{FT} I_o \, \Delta l \, Sa\left(\frac{\omega \Delta l}{2}\right)$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \frac{e^{-JBr}}{r} I_o \, \Delta l \, Sa\left(\frac{\omega \Delta l}{2}\right)$$

(1)

$Sa\left(\frac{\omega \Delta l}{2}\right)$

$$\frac{\omega \Delta l}{2} = \frac{B \cos\theta \, \Delta l}{2}$$

$$= \frac{2\pi}{\lambda} \frac{\Delta l}{2} \cos\theta$$

$$= \frac{\pi}{\lambda} \frac{\lambda}{100} \underbrace{\cos\theta}_{max=1} = \frac{\pi}{100} = 0.0314 \cong 0$$

$$Sa(0) = 1$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \frac{e^{-JBr}}{r} I_o \Delta l$$

\#

Drive expression سؤال